

ON THE REPEATED ERRORS MADE ABOUT KEYNES'S LOGICAL RELATION, V, THE EVIDENTIAL WEIGHT OF THE ARGUMENT, FOR OVER 90 YEARS: FROM BAYLIS (1935) TO BREKEL (2025)

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ABSTRACT

The major error, committed by all academicians writing on Keynes's evidential weight of the argument, V, in Chapter VI of his *A Treatise on Probability*, is the failure to recognize that Probability and Evidential Weight at that point in the treatise are completely independent of each other. They can never be analyzed together until both logical relations have been defined as variables specified in degrees, which are measured by an index defined on the unit interval, $[0,1]$. This means that both probability and evidential weight are defined in terms of degrees.

Keynes's definition of logical probability is $P(a/h) = \alpha$ (Keynes, 1921, p.119), where P denotes the logical relation between propositions a and h. α is a partial degree of rational belief, which is defined on the unit interval $[0,1]$ as $0 \leq \alpha \leq 1$.

Nowhere in chapter VI of Keynes's *A Treatise on Probability* (1921) does Keynes define that V is measured or is defined in degrees. The only type of analysis offered by Keynes in chapter VI is qualitative comparisons involving the logical strength of different arguments based on the relevant evidence.

Only in chapter XXVI does Keynes measure V using an index defined on $[0,1]$, which Keynes called w, the degree of the completeness of the evidence.

Keywords: Decision Weights, Imprecise Probability, Evidential Weight, V, Confidence, Incommensurability, Conventional Coefficient of Weight and Risk, C, W, The Degree of The Completeness of the Evidence

1.0 INTRODUCTION

The paper will be organized in the following manner. Section Two will present many of the errors about Keynes's logical relation, V, in an extensive manner. Section Three will examine the propensity of the current literature to read into Keynes's chapter VI of his TP non-existent "interpretations" that are directly contradicted by a careful reading of the foundation for Keynes's logical analysis of weight, which was based on Boole's relational, propositional logic and argument form as contained in his *The Laws of Thought* (1854, LT, pp.7-8). Keynes's

A Treatise on Probability (TP, 1921) is a continuation of the original logical analysis that was first constructed and applied by George Boole, not J M Keynes. Thus, interval valued probability, objective, logical, probability relations, argument forms, critique of POI, etc., all occurred long before Keynes was born in LT. Keynes, with the help of his father and William E. Johnson, mastered LT and expanded on it in Part II of the TP.

Section Four will examine the many errors of omission and commission contained in Brekel (2022) and Brekel (2025) that were first made in 1935 by Baylis. Section Five will conclude the paper.

Lurking in the background in the literature on Keynes's evidential weight of the argument (ewota) is the shadow of F P Ramsey and the many errors he made about Keynes's logical theory of probability in 1922, 1923 and 1926. Ramsey's errors are also evident in his assessment of Bertrand Russell's handling of the logical foundations of propositions. Ramsey wrote the same kind of unsubstantiated comments about Keynes in January, 1922 in the Cambridge Magazine, as he had done previously when writing about Russell in a November 18th, 1921 Apostle's paper presented a few months earlier. Ramsey's critique was based on Russell's position on the analysis of propositions that Russell held between 1895 and 1908, which were heavily influenced by Moore's Platonic Intuitionism. Russell erroneously treated propositions as if they had been as real, objective, existing objects. Ramsey overlooked that Russell had completely changed his mind regarding the role of propositions in his *The Problems of Philosophy* in 1912, which was due to the impact of Keynes's 1908 Fellowship dissertation on him.

As an example of this influence of Ramsey, consider the following statement by Brekel:

“Unfortunately, Keynes's logical approach to probability faces potentially insurmountable problems. Since the outset, Keynes faced criticisms for his unusual idea that probability arises from a rational assessment of the relation between the premises and conclusion of an argument.² Those criticisms, which began in earnest with Frank Ramsey's review of TP, only gained steam with the rise of the personalist or subjective approach to Bayesian probability³.” (Brekel, 2022, p.1).

In fact, all of Ramsey's claims turn out to be confusions and errors. Consider the following, small subset of Ramsey's total errors:

- There is an “Axiom I” (Ramsey, 1922, p.3) in the *A Treatise on Probability* upon which Keynes theory is based. In reality, such an axiom I does not exist in the TP and is a figment of Ramsey's imagination
- Keynes's argument form is restricted to two propositions. In reality, Keynes's argument form is based on sets of propositions
- A logical probability relation holds between any two given propositions. In reality, Keynes's sets of propositions are restricted to RELATED, NOT RAMSEY'S UNRELATED, propositions
- Keynes claimed that the logical, objective, probability relation is directly perceived through Moorean Platonic intuition. In reality, Keynes uses Boole's definition of intuition, which involves the INTERNAL connections between the h and a propositions

- Keynes's logical, objective, probability relation between propositions does not exist. In reality, Keynes's relation is identical to Boole's logical, probability relation
- Keynes's logical, objective probability relation is based on metaphysical, Platonic forms and relations. In reality, Keynes's relation is identical to Boole's relation
- Keynes's logical, objective probability relation is based on Moore's Platonic Intuitionism, which is based on a single, self-evident proposition. In reality, Keynes's logical relation comes from Boole's *The Laws of Thought* (LT, 1854, pp.7-8), which is based on sets of related propositions.

I could go on indefinitely. (See Brady, 2021 for a systematic examination of Ramsey's many, many other errors made about Keynes's logical theory of probability).

Brekel's belief, that Keynes's logical theory is "an unusual idea", demonstrates that he is ignorant of the fact that Keynes's "unusual idea" is based directly on G. Boole's ideas concerning his construction of the first technical logical theory of probability as presented in LT in 1854.

Section Five will conclude the paper. It will be pointed out that Brekel is simply repeating errors first made by Baylis 90 years ago.

2.0 A CHRONOLOGY OF THE CONTINUING ERRORS MADE IN THE PHILOSOPHICAL, HISTORICAL AND ECONOMICS LITERATURE FROM 1935 TO 2025 ON KEYNES'S EVIDENTIAL WEIGHT OF THE ARGUMENT, V

Baylis made the following false claims in 1935:

"J. M. KEYNES, in his *A Treatise on Probability*,² devotes a chapter to explaining that the weight of the evidence for an assertion of probability, that is, the mere amount of such evidence, ought to be considered in addition to the probability that it confers. To what extent ought to be considered, and the grounds for its importance, he admits he does not know. We are confronted here, he says, with "a very confusing problem."³ As the relevant evidence at our disposal increases, the magnitude of the probability of the proposition on which this evidence bears, may decrease or increase, but in either case something, the amount of the evidence, has increased. "I express this," Keynes says, "by saying that an accession of new evidence increases the weight of an argument."⁴ (Baylis, 1935, p.281).

However, the evidential weight of the argument is a logical relation, not a mathematical function or variable. Baylis was thus the first academician to assert that, for Keynes, $V=K$, where K equals the amount of absolute knowledge. It should have been obvious that a logical relation can't be equal to a number or a mathematical variable. A logical relation can only be deployed qualitatively based on a judgment that the logical strength of argument V1 is greater than the logical strength of argument V2 if V1 is based on more relevant evidence than V2. This conclusion was pointed out by Cohen on page 264 of his 1986 paper.

I J Good then followed this line of erroneous thinking by also setting $V=V(a/h)=K$. This states that the logical relation, the evidential weight of the argument, V, is a number, K, representing the absolute amount of relevant knowledge. (See Brady, 2023 for an extensive analysis of Good's many, many errors made in 40-50 papers on Keynes's chapter VI of the TP over a 40–

50-year period from 1950 to 2000). Of course, nowhere in chapter VI of the TP did Keynes do this because it is mathematically and logically impossible for a strictly qualitative, logical relation to equal a number.

Good was the first to make the claim that V , the evidential weight of the argument, is monotonically increasing. However, the concept of monotonicity or monotonically increasing can only apply to mathematical functions defined by variables or series of numbers, such as $y=f(x)$. V is not a function: It is a logical relation. All that can be said is that the logical strength of one logical relation, relative to its evidence, is greater or lesser than the logical strength of another relation, when the evidence used to compare both arguments has need assessed. We will see that Brekel constantly claims that V is “monotonically increasing” is the hall mark of Keynes’s V relation in chapter VI of the TP. This is simply a gross mathematical and logical error on his part.

Isaac Levi then followed up on Good’s confused contributions by stating that Keynes, in order to measure V correctly, needed to have defined an index in order to measure evidential weight properly. Of course, Levi is correct. However, Levi totally overlooked p.76 of chapter VI of the TP, the footnote on page 76 made to chapter XXVI and chapter XXVI, where Keynes defined just such an index on page 315 to measure V .

Levi argued, following Baylis and Good, that Keynes had then, instead of defining an index to measure V correctly, erroneously set $V(a/h)$ equal to the absolute amount of relevant knowledge, K or

$$V(a/h) = K \text{ (Levi, 1967, pp.141-142).}$$

Of course, nowhere in chapter VI of TP did Keynes ever do this. In fact, such an analysis appears nowhere in anything written by Keynes in his lifetime.

Levi’s work then leads directly to Runde’s work.

Runde implicitly arrives at the conclusion that Good and Levi overlooked that, in fact, Keynes had measured V in three different ways, as

$$V(a/h) = K,$$

$$V(a/h) = K/I, \text{ and}$$

$$V(a/h) = K / (K + I); = 1 / [1 + (I/K)].$$

Of course, nowhere in chapter VI (or chapter XXVI) of TP did Keynes ever do this. Runde commits the same error made by Baylis, Good and Levi, which was to read into chapter VI of the TP his completely unsupported interpretation of V as being a mathematical function represented by variables which can be added or summed. Of course, V is NOT a mathematical function or variable, where number answers can be added or summed, except metaphorically. Again, only Cohen recognized this on p.264 of his 1986 article.

Many hundreds of other authors have repeated the same, exact mistakes by citing Baylis, Good, Levi and Runde over the last 90 years. A few examples are Weatherson (2002), Joyce (2005), and Kasser (2016).

Every single author, dealing with Keynes's logical relation, V , treats V as if it were a mathematical variable, number, function or functor. This includes Levi, Good, and especially Runde and Brekel, as well as Cohen, who contradicted himself in attempting to make Carnap's minor critique of Keynes's second, theoretical definition of irrelevance understandable to non-logicians.

I deal with the latest incarnation of the erroneous work on Keynes's V relation, which is Brekel(2025), in a later section of this paper. Brekel's attempt to read p.315, as well as footnote 2 on that page of Keynes's TP, reveals himself to be very confused about V .

These confusions can be seen in Brekel's claims made about Keynes's index to measure weight, which, as Levi (1967, pp.141-142) realized, is an absolutely necessary requirement in order to measure evidential weight correctly:

"Two main aspects of these passages stand out. First, Keynes seems once again to tie weight to the completeness of our knowledge. Such remarks lend direct textual support to the relative conception of Keynesian weight. Second, Keynes's description of the above coefficient seems to indicate that Keynes intended for weight (at least sometimes) to be indexed to the inclusive interval between zero and one." (Brekel 2025, Section 3.3).

Brekel's first claim, that "Keynes seems once again to tie weight to the completeness of our knowledge "is wrong, as Keynes defines $V(a/h)$ to equal w mathematically. "Seems" has nothing to do with it. Second, Brekel's second claim, that Keynes "... seems to indicate that Keynes intended for weight (at least sometimes) to be indexed to the inclusive interval between zero and one."(Brekel ,2025,Section 3.3) is wrong ,as Keynes defines w mathematically to be in the unit interval $[0,1]$, $0 \leq w \leq 1$, at all times ,given the normalization condition, $K + I = 1$.Brekel's "seems" characterization of Keynes's mathematical work demonstrates that he does not understand what mathematical reasoning and modelling involves.

Brekel's claim, that "...Keynes intended for weight (at least sometimes) to be indexed..." leads one to conclude that Brekel has very severe problems reading English, as well as exhibiting severe mathematical deficiencies in understanding the mathematics involved in specifying an index of weight. Such an index must hold all of the time, not some of the time.

Let us now consider the contributions of Bill Gerrard.

Consider the following erroneous definition of weight by Gerrard, given that $V=V(a/h)$:

"The weight of argument, V , is a measure of the amount of evidence, h , on which the proposition, a , is based." (Gerrard,2003, p.240; see also p.241 for the continuation of Gerrard's erroneous discussion of V .)

Gerrard does not understand that V is a logical relation. It can measure nothing. It is only on p.315 of chapter XXVI of the TP where Keynes defines that V is measured by w , where w is

defined on the unit interval $[0,1]$ as $0 \leq w \leq 1$, so that $V(a/h) = w$. It is w which is a measure of the completeness of the amount of evidence

Gerrard has failed to realize that Keynes's Evidential Weight of the Argument, $V(a/h) = w$ is, like Keynes's $P(a/h) = \alpha, 0 \leq \alpha \leq 1$, a logical relation defined by Keynes on p.119 of the *A Treatise on Probability* as being measured by w (p.315), where $0 \leq w \leq 1$. Gerrard repeats the mathematical and logical errors regarding Keynes's V originally made by Baylis, Good, Levi and then made again by Runde (1990). [See Brady ,2022, pp. 64- 65, for the correction of Runde 's errors].

Weatherson (2002) is another example of a philosopher who never read Part II of Keynes's TP and substituted Ramsey's critique for Keynes's analysis. Weatherson based his critique of Keynes heavily on Ramsey's many erroneous claims. Consider Weatherson's characterization of Keynes's non numerical (Boolean, interval valued, indeterminate probabilities) probabilities:

"In the *Treatise on Probability* (Keynes (1921), hereafter TP) Keynes says that probability is essentially a property of ordered pairs of propositions, or what he calls arguments. He writes $p/q = \alpha$, for the probability of hypothesis p on evidence q is α . Now this value α is rather unusual. It sometimes is a number, but sometimes not; it sometimes can be compared to all numbers, but sometimes not; it sometimes can be compared to other probability values such as β , but sometimes not and it can enter into arithmetic operations. As a consequence, probabilities are subject to all the usual rules of the classical probability calculus. For example, whenever p and r are inconsistent, then $(p \vee r)/q = p/q + r/q$ always holds, even when none of these values is numerical.

These five properties are rather perplexing. Indeed, Keynes's failure to explain or justify them fully is one of the main criticisms that Ramsey (Ramsey, 1926, 161-6) launches at Keynes's theory. ...Ramsey is right to question the intelligibility of Keynes's use of addition and multiplication. We know what it means to add and multiply numbers, but we have no idea what it is to add or multiply non-numerical entities." (Weatherson,2002, p.5).

First, Keynes's Boolean based theory deals with sets of h (premises)propositions and a (conclusions) propositions in an argument form which is analyzed by applying Boole's relational, propositional logic. Nowhere in Keynes's TP does Keynes state that he is dealing just with ordered pairs of propositions, as originally was erroneously claimed to hold by Cohen (1986, p.246). Weatherson does not cite Cohen.

Second, due to Weatherson's complete ignorance of Keynes's application of Boolean, interval valued probability in Part II of the TP, Weatherson makes up what he calls "...five properties ..." which do not exist in the TP, but only in Weatherson's imagination(nowhere in his article does Weatherson cite a single page of Keynes's book where these five properties appear) .This approach is identical to the fictional and fictitious definitions of axioms made by Ramsey that do not exist in Keynes's book.

Third, everything written by Ramsey in 1922 and 1926 on Keynes's non numerical(interval), Boolean probabilities make no sense because Ramsey never understood what an interval valued probability was.

Continuing, Weatherson repeats Cohen's 1986 errors, which involved attempting to integrate Keynes's logical, qualitative discussions of V in chapter VI of the TP into a Bayesian, conditionalization application /analysis which it is impossible to perform because Keynes had not yet provided any defined measurement of V , which occurs only in chapter XXVI on page 315 and is applied by Keynes only in footnote 2 of page 315:

“The simplest definition of relevance is that new evidence e is irrelevant top given old evidence q iff $(p/q \wedge e) = p/q$, and relevant otherwise. Now there is a problem.

Two pieces of evidence e_1 and e_2 can be irrelevant taken together, but relevant taken separately. For a general example, let e_1 be $p \vee r$ and e_2 be $\neg p \vee r$, for almost any proposition r . If I receive e_1 and e_2 sequentially, the weight of the argument from my evidence top will have increased twice as I receive these new pieces of evidence.

So it must be higher than it was when I started. But if I just received the two pieces of evidence at once, as one piece of evidence, I would have properly regarded it as irrelevant. Hence the weight in question would be unchanged. So it looks as if weight depends implausibly not on what the evidence is, but on the order in which it was obtained.

Keynes avoids this implausibility by tightening up the definition of irrelevance. He says that e is irrelevant to p/q iff there are no propositions e_1 and e_2 such that e is logically equivalent to $e_1 \wedge e_2$ and either e_1 or e_2 is relevant to p/q . Unfortunately, as I noted in the previous paragraph for virtually any such evidence proposition there will be such propositions e_1 and e_2 . This was first noticed by Carnap.” (Weatherson,2002, p.6-Note that Weatherson's claim that what Carnap did was “simple “is false. Carnap was the first to notice, as well as to provide a detailed critique, based on the correct assumption that Keynes never defined and measured e wota in chapter VI of the TP. Carnap is correct, relative to his implicit assumption that Keynes never explicitly defined and measured weight, in chapter VI. However, his minor correction of Keynes's stronger definition of irrelevance in no way undermines Keynes's standard use of his relevance and irrelevance definitions in his applied theory of evidential weight in chapter VI:

“Keynes has given definitions for the concepts of irrelevance and relevance which we have adopted (except for the somewhat wider sense which we have given to 'irrelevance' in D6s-rd). He believes, however, that another, stronger concept of irrelevance would be theoretically preferable. ([Probab.], p. 55). His definition for it, expressed in our terminology, is as follows: i is irrelevant in the strict sense to h on evidence $e = Df$ there is no j such that $c(j, e, i) = 1$, $c(j, e) \neq 1$, and $c(h, e, j) \sim c(h, e)$.” (Carnap,1950, pp.419-20; underline added).

Note that Carnap does not disagree with Keynes's standard definitions of relevance and irrelevance, but only with Keynes's other ,stronger theoretical definition of irrelevance .Keynes's approach in chapter VI to evidential weight works fine with his original definition of irrelevance .Unfortunately, Carnap overlooked chapter XXVI .Keynes's definition ,that the ewota is defined and measured by w , the degree of the completeness of the relevant data(knowledge) upon which the probabilities are based, essentially eliminates irrelevant data/knowledge from being any part of the degree of the completeness , w .

However, it must be mentioned here that his handling of Keynes's “probable” (standard) error in his chapter nine of Logical Foundations (1950) is excellent, so long as one is talking about

applications in physics, engineering physics, engineering, electronics, biology and chemistry and not social sciences, behavioral sciences or liberal arts).

All of this analysis is very similar to that on pp.267-268 of Cohen (1986), who is not cited in Weatherson's article.

Weatherson then arrives at the following conclusion:

“Keynes's theory of probability is based around some non-numerical values whose nature and behaviour is left largely unexplained, and a concept of weight which is subject to a telling and simple objection. Nevertheless, his core ideas, that probabilities can but need not be precise, and that we need a concept like weight as well as just probability, both seem right for more general reasons. Hence the theory here, which captures the Keynesian intuitions while explaining away his mysterious non numerical values and making the concept of weight more rigorous, looks to be as good as it gets for a Keynesian theory of uncertainty.” (Weatherson,2002, p.7).

We can correct Weatherson's many errors here, since it is simply false that

“Keynes's theory of probability is based around some non-numerical values whose nature and behaviour is left largely unexplained, and a concept of weight which is subject to a telling and simple objection”

and false that we need to explain away

“.. his mysterious non numerical values and making the concept of weight more rigorous.”

Keynes makes the concept of evidential weight of the argument much stronger in chapter XXVI by quantitatively measuring V by defining an index, w . He then integrates V , which is measured by the mathematical variable w , so as to combine it in a manner that provided a decision theoretical approach to the analysis of risk(probability) and weight (uncertainty, ambiguity) combined, which was the first such approach made in history.

Keynes explains precisely what non numerical values are in Part II of the TP. They are Boolean interval valued probabilities, which Boole called indeterminate. Of course, Weatherson never read Parts II, III, IV or V of the TP before he published his 2002 article in the CJE, which accounts for his erroneous claims about Keynes's(a) non-numerical probabilities and(b) evidential weight.

Keynes finished his theory of evidential weight in chapter XXVI of the TP by providing a quantitative measure of V , which excludes the problem discussed first by Cohen and then, in nearly identical fashion, by Weatherson.

Note that Cohen also “fixes” Keynes's “problem “involving his stronger definition of irrelevance on page 268 as it relates to Keynes's stronger theoretical definition of irrelevance in chapter VI. This is not mentioned by Weatherson.

Let us now consider Joyce (2005):

“Let us first distinguish between the balance of the evidence, which is a matter of how decisively the data tells for or against the hypothesis, and what J.M. Keynes (1921) called the weight of the evidence, which is a matter of the gross amount of data available. Here is Keynes:

As the relevant evidence [for a hypothesis] at our disposal increases, the magnitude of [its] probability may either decrease or increase, according as the new knowledge strengthens the unfavorable or favorable evidence; but something seems to have increased in either case—we have a more substantial basis on which to rest our conclusion... New evidence will sometimes decrease the probability of [the hypothesis] but will always increase its ‘weight’. (1921, p.77).” (Joyce,2005, p.158).

Keynes’s ‘weight’ is short for the term “evidential weight of the argument “, V , which is a logical relation. It has nothing to do with “... the gross amount of data available. “Keynes’s discussions in chapter VI of the TP are qualitative only. There is no quantitative measure specified in chapter VI, as pointed out by Cohen (1986). The few additional comments made by Joyce on Keynes are incorrect, as Keynes does exactly what Joyce claims he never did—give a precise mathematical, quantitative measure for V .

Let us now move on to Basili and Zappia (2009). They claim the following, which is simply a restatement of Runde in 1990:

“There is a substantial literature about the Keynesian notion of weight of argument. Possibly because of Keynes’s inability to adhere to a precise definition, the weight has been given different readings... Runde (1990) was probably the first commentator to call attention to the fact that there are different definitions of weight in the Treatise. Runde emphasized the importance of the definition of evidential weight as the degree of completeness of information on which a probability assessment is based, rather than the mere absolute amount of evidence implicit in the second order probability interpretation. In Keynes’s approach, that is, new evidence can increase the relevant ignorance more than the relevant knowledge, thus decreasing the weight of argument.¹³” (Basili and Zappia,2009, p.424).

Footnote 13 gives Runde’s three conflicting definitions:

“¹³ Runde (1990) has proposed the following notation for the different notions of weight (V) of a certain proposition (a) given the available evidence (h), in terms of knowledge (K) and ignorance (I): absolute amount of relevant knowledge: $V(a/h)=K$; balance of absolute amounts of relevant knowledge and ignorance: $V(a/h)=K/I$; and degree of completeness of information: $V(a/h)=K/(K + I)$. As Keynes (1921, p. 348) stated that the weight can be measured by means of the closed interval between 0 and 1, assuming $K + I = 1$ only the third definition of weight seems accurate.” (Basili and Zappia,2009, p.424)

Pace Basili and Zappia, Keynes gave only one definition of V . It is $V=V(a/h) = w$, where $0 \leq w \leq 1$ and $w=K/(K+I)$. This definition can be found on page 315 of the 1921 edition of the TP.

Consider the following statement by Kasser:

“In his 1921 *A Treatise on Probability*, Keynes writes:

As the relevant evidence at our disposal increases, the magnitude of the probability of the argument may either increase or decrease, according as the new knowledge strengthens the unfavourable or the favourable evidence; but something seems to have increased in either case, —we have a more substantial basis upon which to rest our conclusion. I express this by saying that an accession of new evidence increases the weight of an argument. New evidence will sometimes decrease the probability of an argument, but it will always increase its 'weight.' (p. 71)

Keynes contrasts the balance metaphor and the weight metaphor, and clearly has gross, not net weight of evidence in mind. As we will see below, some philosophers have treated Good's notion of weight of evidence as a competitor to Keynes'.

Others have held that it competes with standard measures of degree of confirmation like $P(H/E)$ — $P(H)$ or $P(H\setminus E)/P(H)$.¹⁸ Good seems to think he has formulated a conception that, in one stroke, improves upon these two measures of confirmation and on the Keynesian notion of evidential weight.¹⁹ This is odd because what Keynes means by weight of evidence isn't a competing conception of the balance of evidence but instead something quite different. He is trying to quantify the amount of evidence relevant to the conclusion, not the amount of support provided for the conclusion. So Keynesian weight, unlike the Turing-Good notion, is independent of probability.²⁰ Like Peirce, Keynes thinks that at least two numbers will be needed.” (Kassar,2016, pp.639-640)

Kasser makes some good points here. However, on the fundamental point concerning measurement, Kasser overlooks that nowhere in chapter VI of the TP is there any attempt to measure V by Keynes. He does that in chapter XXVI only.

3.0 The major problem inherent in reading the literature on Chapter VI of Keynes's A Treatise on Probability is that the authors are looking for a discussion of measurement that does not exist

Cohen correctly specified Keynes's Chapter VI method of analysis in his following statement from his 1986 article, which, despite the later errors he made in his article dealing with (a) the integration of evidential weight into Bayesian conditionalization and (b) his failure to integrate chapter XXVI into his analysis, remains, by far, the best discussion of Keynes's evidential weight, V , in the literature since 1935:

“Keynes says that, metaphorically, the weight of the argument from E to H measures the sum of the favourable and unfavourable evidence that E states for H , and the probability measures the difference. But he does not suggest any method by which weights might be measured and in fact admits that often one cannot even compare the weights of different arguments. He thinks that 'in deciding on a course of action, it seems plausible to suppose that we ought to take account of the weight as well as the probability of different expectations'. But he finds it difficult to think of any clear example of this...” (Cohen,1986, p.264).

Cohen is correct in all of the points he is making above, as opposed to all other writers on Keynes's V since 1935, that in chapter VI, Keynes does not ever

“.... suggest any method by which weights might be measured and in fact admits that often one cannot even compare the weights of different arguments. “

The reason for this is that the logical probability is measured in degrees while the evidential weight relation is not measured in degrees. Therefore, they are incommensurable as far as Keynes's chapter VI analysis goes.

Of course, Keynes solves this fundamental problem in chapter XXVI, which follows from his discussion and footnote on p.76 (footnote 1 on p.76) of the *A Treatise on Probability*, which has been ignored /overlooked/not read/forgotten by all academicians writing on Keynes's analysis of evidential weight in chapter VI of the *A Treatise on Probability* from Baylis to Brekel.

In Chapter 26, Keynes provides a way to meet Cohen's criticism, that

“He thinks that, ‘in deciding on a course of action, it seems plausible to suppose that we ought to take account of the weight as well as the probability of different expectations. But he finds it difficult to think of any clear example of this...’ (Cohen, 1986, p.264).

Keynes does this by measuring the evidential weight of the argument by w , which Keynes defines as the degree of the completeness of the relevant information upon which the probability is based. Keynes starts by setting $K + I = 1$, so that V can be measured in degrees. Then he specifies that

$V = V(a/h) = w$, where $0 \leq w \leq 1$ and $w = K/(K+I)$. The complement of w is $1-w$.

Keynes has now defined and measured both probability and evidential weight in degrees:

$P(a/h) = \alpha$, where $0 \leq \alpha \leq 1$ and

$V(a/h) = w$, where $0 \leq w \leq 1$.

Only now can both probability and weight be combined in an analysis that incorporates both in a new mathematical, quantitative expectations approach that Keynes calls the “conventional coefficient of weight and risk, c ”. The goal is to maximize cA , where A can stand for expected value or expected utility, instead of pA alone. c equals the following:

$c = 1/(1+q) [2w/(1+w)]$.

$1/(1+q)$ and $[2w/(1+w)]$ are decision weights that allow one to convert interval valued probability to numerical probability subject to constraints. $1/(1+q)$ deals with probability preferences, taking into account nonlinear probability preferences, as opposed to the assumption of linear probability preferences specified in the purely mathematical laws of the probability calculus. $2w/(1+w)$ takes into account non additivity, which occurs if $w < 1$. The standard assumption of the laws of the mathematical theory of probability is that w ALWAYS is equal to 1, so that you always have a complete ordering of the probability space and a complete sample space of all possible outcomes before any decision has to be made from a set of the different choice options.

I have covered Keynes's coefficient model and index to measure V by w repeatedly in 6 articles published in Psychological Reports between 1989 and 1994, in Synthese in 1987, British Journal for the Philosophy of Science in 1993, International Studies in the Philosophy of Science in 1994, five articles in International Journal of Applied Economics and Econometrics in 2001, 2002, 2011, 2012(a, b), three articles in Theoretical and Practical Research in Economic Fields in 2023 and 2024(a, b), and an additional 125 articles published at SSRN, ResearchGate and Academia.com between 2009 and 2025.

Finally, my two 2004 books, titled "Essays on John Maynard Keynes and" and "J.M. Keynes's Theory of Decision Making, Induction, and Analogy", cover the many, many errors made about Chapter VI and XXVI of the A Treatise on Probability in a number of chapters, as well as specifically in chapter 17, pp.379-408 & chapter 20, pp.444-462 in the former book and in chapters 5 & 6, pp.100-147 of the latter book. Both chapters specifically examine the errors of Runde in his 1990 article, titled "Keynesian Uncertainty and the Weight of Arguments.", in which Runde claimed that Keynes had presented three different ways to measure V , which is impossible, as one can't measure a logical relation unless one first normalizes on the unit interval $[0,1]$ first.

It has been demonstrated that every author, who has written on Keynes's evidential weight, be it in economics, history, philosophy, social science, behavioral science, law, mathematics or statistics in the period 1935 to 2025, simply kept on repeating the mistakes made by the authors in earlier time periods, such as Baylis and Good, of trying to find a discussion of measurement in Chapter VI that is non-existent. Every author, writing on Keynes's logical relation, V , completely ignored the basic independence problem impacting any analysis of probability and evidential weight in chapter VI of the TP. This type of error demonstrates a complete lack of understanding of Keynes's correct mathematical and logical analysis contained in chapter XXVI by claiming that Keynes was trying to measure V in chapter VI, which Keynes realized was an impossibility, as he had not yet defined an index to measure V with. A small sample of the worst offenders, all major contributors to the literature on Keynes's logical relation, V , are

- Baylis (1935)
- Good (1950, etc.)
- Levi (1967)
- Runde (1990)
- Gerrard (2003)
- Weatherson (2002)
- Joyce (2005)
- Basili, Zappia (2009)
- Kasser (2016)
- Brekel (2022)
- Brekel (2025)

However, there are literally many hundreds of other publications that cite the above list as their references, excluding Brekel. This means that the confused and confusing contributions on the topic of Keynes's evidential weight of the argument are passed on to a new generation of authors. Over time, there is a proliferation of articles, short articles, book reviews, comments,

replies, rejoinders, encyclopedia articles , handbook articles , books, book chapters, collections of essays, working papers, etc., which are all based on the erroneous work listed above.

Lurking in the background of all of these authors' works is their misbelief that F P Ramsey had demonstrated the existence of severe logical, epistemological, philosophical and methodological errors in Keynes's work in his January, 1922 Cambridge Magazine review and his 1926 "Truth and Probability" presentation, which was published in 1931. In fact, Ramsey's "demonstrations" of supposed errors, that he claimed existed in Keynes's logical theory of probability, are the result of delusions, illusions and hallucinations on Ramsey's part. The claims made by Ramsey are based on fictitious, nonexistent and fictional materials that were dreamed up by Ramsey in his imagination and do not exist in Keynes's *A Treatise on Probability*. Bertrand Russell easily and completely obliterated these fictional claims in one small * footnote presented on page 120 of his review of Keynes's book in the July, 1922 issue of the *Mathematical Gazette*. He showed that Ramsey's "demonstrations" always involved specifying pairs of unrelated propositions that Boole and Keynes had automatically excluded from the Boolean, relational, propositional logic that was required to deal only with sets of related (relevant) propositions.

Brekel's 1922 claims, about "A very confusing problem: interpreting Keynesian weight", are based on the confused and confusing work of the authors writing on Keynes's *V* relation who went before him, many of which he read, cited, and has based all of his 2022 and 2025 contributions on.

4.0 Brekel in 2022 and 2025

We will first deal with Brekel, 2022 and then with Brekel, 2025. J. Brekel's 1922 master's thesis is titled,

"A very confusing problem: Interpreting Keynesian weight".

However, there was no confusion on Keynes's part in his analysis of the evidential weight of the argument, *V*. Chapter VI of the TP provides a logical analysis, while chapter XXVI provides a mathematical analysis. Brekel is the one who is very confused. He is confused because he has based his thesis on the works of those who went before him; thus, he simply repeats all of their mistakes again and again. The crucial error made throughout Brekel's thesis is to treat Keynes's logical relation, *V*, $V=V(a/h)$, as if it were a function like $y=f(x)$, a single number or as a series of numbers which can be added sequentially. A logical relation is not a function. It is not a number. It can't be measured UNLESS it is first converted into degrees, in the same way that probability is measured in degrees. One MUST define an index defined on the unit interval $[0,1]$ by normalization. Keynes did not do this in Chapter VI of the TP as he intended to devote chapter VI strictly to a logical analysis of *V*. One can thus talk about, in some cases only, where one can make a judgment about the increasing logical strength of the conclusion as more relevant evidence is incorporated in the premises or state that the logical strength of one argument is greater than the logical strength of another, comparable argument. Talk about measuring weight or adding or summing evidence is strictly metaphorical. Keynes fully warned readers of this on p.76 and in a footnote on page 76 made to chapter XXVI of the TP. He stated that he would later provide a quantitative and mathematical analysis in chapter

XXVI, when the question arose as to how do combine both weight and risk so as to discuss applications of decision making under BOTH risk and uncertainty. This analysis complements the strictly qualitative analysis provided about V in chapter VI. Of course, everyone writing on Keynes's chapter VI in the 20th and 21st centuries skipped the analysis of w and the c coefficient, contained in chapter XXVI, and then read into chapter VI their own private, subjective thoughts, "interpretations" and ideas about what they believed Keynes must have meant. The "interpretation "approach to studying Keynes is the result of the failure to read Keynes's A Treatise on Probability. Since each author writing on Keynes's unread book incorporates his own ideas, you get a proliferation of conflicting claims about "What Keynes MUST have meant" about weight. This approach is due to the inability of readers of Keynes's works to understand Keynes's approach to mathematical, logical, and statistical modeling. It is an imprecise and inexact one emphasizing approximation in quantitative modeling, and not exact and precise techniques. The best examples of this severe confusion regarding Keynes's application of logical, mathematical and statistical analysis are the works on Keynes written by Ramsey, Good and Tinbergen, who all believed in precise and exact mathematical analysis, where the only answers are single, numerical answers. Their critiques of Keynes contain literally hundreds of major and minor errors. Brekel is thus in good company.

Brekel does understand that Keynes's ewota and probability are independent of each other (Brekel,2022, pp.25-26 and 36-37), but has no idea about how this impacts an analysis of ewota, as he never mentions p.76 and the connected footnote on p.76 directing the reader to chapter XXVI, in his thesis.

There is nothing in Brekel(2022) which is new, novel, original, unique, creative or innovative that is correct. Only a few of Brekel's errors will be covered here.

First, Brekel claims, correctly, that

"As we saw above, Keynesian weight is traditionally construed as a function of the amount of evidence in question, which explains the name of gross weight." (Brekel,2022, p.29).

Of course, it is mathematically impossible for the logical relation of evidential weight, V, to be a function (or functor). That means that all authors who proceeded Brekel, except Cohen, who clearly stated that there is no measurement being made in chapter VI, are dead wrong, as V is never measured in chapter VI of the TP.

Second, Brekel claims, incorrectly, that

"Ultimately, I think a careful reading of Good's critique demonstrates the way in which Keynesian weight remains a measure of gross

weight, ..." (Brekel,2022, p.30).

This is quite impossible, as there is only one place in the TP where Keynes measures evidential weight, V. This occurs in chapter XXVI on p.315. Keynes measures V by w, which is defined as

$$V=V(a/h) =w,$$

where w is defined on the unit interval $[0,1]$, as Keynes has normalized by setting $K + I = 1$, so $0 \leq w \leq 1$, where $w = K/(K+I)$ and $1-w = I/(K+I)$. Keynes's definition of V has the same form as

$$P = P(a/h) = \alpha, 0 \leq \alpha \leq 1.$$

Note that for applications of his conventional coefficient, c , Keynes uses p to denote probability of success instead of α , which is normalized on the unit interval $[0,1]$, so $1-p=q$.

Given the above fact, it is easy to dismiss Brekel's claim that

“Based on the last paragraph in the above quote from Good, it seems as though Good interpreted measuring total accumulated evidence as the only function of Keynesian weight. Consequently, Good operates with a monotonic interpretation of Keynesian weight in mind. Recall that on the monotonic interpretation, Keynesian weight increases with any additions to the evidence involved in the probability judgement, regardless of how the additional evidence affects the probability judgement.¹¹⁸ Accordingly, monotonic Keynesian weight only provides a single, absolute value for the weight of a probability judgement. Since Keynesian weight is meant to measure the gross amount of evidence incorporated into a probability judgement, using a single number makes sense.” (Brekel, 2022, p.62)

Actually, It makes nonsense, not sense. It is impossible to measure V , a logical relation as being (a) a function, (b) a number or (c) a monotonically increasing function, or series or sequence of numbers.

Brekel (2020) spends pages attempting to deal with Runde's “interpretation” involving three different definitions that are purported to represent Keynesian weight.

I had already shown, for instance in articles published in my two 2004 books, that pp.279-283 of Runde's 1990 paper “Keynesian Uncertainty and the weight of the arguments”, that contain Runde's claims, are completely wrong, as it is mathematically and logically impossible to measure a logical relation by a number or a function.

We now will move to evaluate Brekel (2025).

Let us first refer back to Cohen's excellent summary concerning chapter VI of the TP before getting into all of Brekel's many confusions:

“Keynes says that, metaphorically, the weight of the argument from E to H measures the sum of the favourable and unfavourable evidence that E states for H , and the probability measures the difference. But he does not suggest any method by which weights might be measured and in fact admits that often one cannot even compare the weights of different arguments. He thinks that, 'in deciding on a course of action, it seems plausible to suppose that we ought to take account of the weight as well as the probability of different expectations'. But he finds it difficult to think of any clear example of this...” (Cohen, 1986, p.264; underline added).

Cohen is clear, although later in his article he made some statements that directly contradict his clear message on p.264, that, because V is a logical relation which, as Levi pointed out, can ONLY be measured by specifying an index by normalization on the unit $[0,1]$, then

- Any talk by Keynes in chapter VI about the measurement of weight is strictly metaphorical
- Keynes does not discuss any method of measuring weight in chapter VI of the TP
- Keynes states only that some comparisons of weights can be made.

Consider Brekel's section 3," Runde's Interpretation:3.1 Sheer Weight"

"The most straightforward conception of Keynesian weight comes via "sheer Keynesian weight." Sheer Keynesian weight simply measures the absolute amount of relevant evidence on which a probability relation is based. This conception is sometimes called "monotonic Keynesian weight" because on this reading, a gain in relevant evidence must (by definition) increase Keynesian weight. It is impossible for sheer Keynesian weight to decrease when learning new relevant evidence. To draw an analogy to our everyday, physical notion of weight, the sheer conception of Keynesian weight treats the addition of relevant evidence similarly to the addition of mass while holding the force of gravity constant—additional mass always increases weight. We can formally represent sheer Keynesian weight as the following:

$$V(c|e) = K$$

Note that the above representation does not appear anywhere in Keynes's own work (italics added). Instead, Runde uses this formalization to provide a useful framework for thinking about the characteristics of sheer Keynesian weight. Above, $c|e$ represents the argument and, in keeping with Keynes's own choice of notation, V represents sheer Keynesian weight." (Brekel,2025, p.7; underline added).

There is a reason that none of Runde's "interpretation "appears in chapter VI of the TP or in anything written by Keynes in his lifetime. Runde's "interpretations "are, like Ramsey's "interpretations", hallucinations that do not exist in the TP. The entire discussion by Brekel is erroneous. It is in direct conflict with all of Keynes's analysis contained in chapter VI, as well as with the three points made by Cohen above, which Brekel claimed he read.

$V=V(a/h)$ means that V is a logical relation connecting the premises and conclusion of an argument, as defined by Boole (1854, pp.7-8). It is impossible for V to represent "sheer Keynesian weight." (Brekel,2025, p.7).

Brekel repeats his erroneous approach in Section 3.2, titled "Relative Weight:

"Runde gleans two non-monotonic conceptions of weight from Keynes's work (Runde, 1990, 280–1) ..." (Brekel,2025, p.8).

Continuing, Brekel states the following:

"With that initial characterization in mind, consider the following formal representation of relative weightodds :

$$"W(c|e) = K/I.$$

Once again, note that the above representation does not appear in Keynes's work. "(Brekel,2025, p.9; italics added).

It should now be obvious, even to a severely confused author such as Brekel, that Runde is simply making things up as he goes along. Runde, much like Ramsey and Good before him, is not interpreting Keynes's analysis; Runde is simply methodically making it all up in his own mind. This equation can't possibly appear in Keynes's work because nowhere in chapter VI of the TP, except to warn the reader that references to measurement in chapter VI about weight are metaphors, does Keynes measure weight. I will again repeat Cohen's correct summary of what is happening in chapter VI of the TP:

"Keynes says that, metaphorically, the weight of the argument from E to H measures the sum of the favourable and unfavourable evidence that E states for H, and the probability measures the difference. But he does not suggest any method by which weights might be measured and in fact admits that often one cannot even compare the weights of different arguments..." (Cohen,1986, p.246)

Finally, Brekel goes to chapter XXVI to obtain Runde's third version, which he writes as

" $Cw(c/e)=K/(K+I)$."(Brekel,2025, p.9)

Brekel does not inform his readers that Runde (a)rejected Keynes's own discussion of the measurement of weight on p.315 and footnote 2, as regards Keynes's c coefficient, as well as Runde's claim that b) $K/(K+I)$ can be divided by K to obtain

$K/K/(K+I)/K=1/(1+w)$, where $w=I/K$. It is completely unclear what it was Runde thought he was doing here, as it has absolutely nothing to do with Keynes's defining an index, normalized on the unit interval $K+I=1$, so that $V=V(a/h)=w$, where $0 \leq w \leq 1$, $w=K/(K+I)$, where the complement of w, $1-w$, is equal to $I/(K+I)$.

We now arrive at Brekel's attempt to deal with (a) Keynes's specification of an index to measure evidential weight, which, as pointed out by Levi, is the only CORRECT way to measure the evidential weight of the argument, as well as all other similar measures that came after Keynes, such as Ellsberg's rho, ρ .

Brekel's attempts to read p.315 of Keynes's TP reveals himself to be very confused about Keynes's correct mathematical analysis. This conclusion can be seen as an accurate description of Brekel's claims, made about Keynes's index to measure weight, which, as Levi (1967, pp.141-142) realized, is an absolute necessary condition in order to measure weight, as well as in Keynes's c coefficient:

"Two main aspects of these passages stand out. First, Keynes seems once again to tie weight to the completeness of our knowledge. Such remarks lend direct textual support to the relative conception of Keynesian weight. Second, Keynes's description of the above coefficient seems to indicate that Keynes intended for weight (at least sometimes) to be indexed to the inclusive interval between zero and one." (Brekel 2025, Section 3.3).

Brekel's first claim, that "Keynes seems once again to tie weight to the completeness of our knowledge overlooks the mathematical definition of w . Keynes defines $V(a/h)$ to equal w mathematically, where $0 \leq w \leq 1$. "Seems" has nothing to do with it. Second, Brekel's second claim, that Keynes "... seems to indicate that Keynes intended for weight (at least sometimes) to be indexed to the inclusive interval between zero and one. "Again, Brekel's use of the word "seems" has nothing to do with anything. Brekel's Section 3.3 makes no sense, as Keynes mathematically defines w on the unit interval $[0, 1]$, $0 \leq w \leq 1$, at all times, given the normalization condition, $K + I = 1$.

Brekel's "...Keynes intended for weight (at least sometimes) to be indexed..." fully supports a characterization of Brekel as being mathematically confused.

I will devote the rest of my examination to Brekel's continuing series of erroneous claims made about measuring weight in chapter VI of the TP.

Consider Brekel's continuing failure throughout his paper to recognize that NO MEASURE of V is defined, discussed or applied in chapter VI of the TP:

"Although this aspect of the text appears to support Runde's interpretation, numerous other parts of the text clearly conflict with Runde's relative conception of weight. Contrary to the relative conception of weight, Keynes consistently says that weight must increase with new relevant evidence. For instance, Keynes says,

New evidence will sometimes decrease the probability of an argument, but it will always increase its "weight" (Keynes, 1921, 78, emphasis added).

Keynes also says,

If the new evidence is "irrelevant," in the more precise of the two senses defined in §14 of Chapter IV., the weight is left unchanged. If any part of the new evidence is relevant, then the value is increased...Starting, therefore, with minimum weight, corresponding to a priori probability, the evidential weight of an argument rises, though its probability may either rise or fall, with every accession of relevant evidence (Keynes, 1921, 78–9), as well as, if we are to be able to treat "weight" and "relevance" as correlative terms, we must regard evidence as relevant, part of which is favourable and part unfavourable, even if, taken as a whole, it leaves the probability unchanged. With this definition, to say that a new piece of evidence is "relevant" is the same thing as to say that it increases the "weight" of the argument (Keynes, 1921, 79).

Additionally, Keynes's second rule for comparing weights implies that weight always increases with increases in relevant evidence (Keynes, 1921, 80). Keynes is repeatedly clear on this point—monotonicity is a fundamental feature of weight. Keynes reiterates the point later in Chapter VI:

The fundamental distinction of this chapter may be briefly repeated. One argument has more weight than another if it is based upon a greater amount of relevant evidence; but it is not always, or even generally, possible to say that of two sets of propositions that one embodies more evidence than the other. It has a greater probability than another if the balance in its favour, of what evidence there is, is greater than the balance in favour of the argument with

which we compare it; but it is not always, or even generally, possible to say that the balance in one case is greater than the balance in the other. The weight, to speak metaphorically, measures the sum of the favourable and unfavourable evidence, the probability measures the difference.” (Keynes, 1921, p. 85.-Author’s note- Note that this pagination is to the CWJMK 1973 edition of the TP and not the 1921 edition).

Throughout his primary explication of weight, then, Keynes seems emphatic about weight increasing with increases in relevant evidence.” (Brekel,2025, pp.13-14).

Brekel still does not grasp that the evidential weight of the argument, V , is a logical relation. A logical relation can’t increase or decrease, etc. All the quotes presented above from chapter VI of the TP by Brekel are metaphoric.

Here is the only thing that you can say about V in chapter VI of the TP. The logical strength of the evidential weight of the argument, V , that holds between the premises and conclusion of an argument, becomes stronger as more evidence is added in the premises. Period.

Here is what you cannot say about V , but only about w , after Keynes’s chapter XXVI analysis of the TP is grasped. The degree of completeness of the relevant evidence, w , increases as more relevant evidence has been obtained. Therefore, contrary to Brekel and everyone else who has written on Keynes’s logical relation, V , it is not V that is increasing with increases in relevant evidence, as claimed by Brekel:

“Throughout his primary explication of weight, then, Keynes seems emphatic about weight increasing with increases in relevant evidence. Accordingly, Runde’s interpretation conflicts with a large swath of textual evidence. Indeed, Runde acknowledges this issue by noting that his interpretation “contradicts many of Keynes’s other statements” (Runde, 1990, 283).” (Brekel,2025, p.14)

Brekel nowhere is able to come to the only conclusion possible, which is that It is w that is increasing or decreasing. V can’t increase, decrease or remain constant. It is not a number, a sequence or series of number, a variable, function or functor. w can equal a number and is a mathematical variable

Another error by Brekel occurs with regards to Keynes’s argument form, $P(a/h)$. The argument form has the same, identical (a/h) propositional meaning that appears in $V(a/h)$. Keynes makes it clear that knowledge is subjective, not objective, to the individual decision maker:

“All propositions are true or false, but the knowledge we have of them depends on our circumstances; and while it is often convenient to speak of propositions as certain or probable, this expresses strictly a relationship in which they stand to a corpus of knowledge, actual or hypothetical, and not a characteristic of the propositions in themselves. A proposition is capable at the same time of varying degrees of this relationship, depending upon the knowledge to which it is related, so that it is without significance to call a proposition probable unless we specify the knowledge to which we are relating it.

To this extent, therefore, probability may be called subjective. But in the sense important to logic, probability is not subjective. It is not, that is to say, subject to human caprice. A

proposition is not probable because we think so. When once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively, and is independent of our opinion. The Theory of Probability is logical, therefore, because it is concerned with the degree of belief which it is rational to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational.

Given the body of direct knowledge which constitutes our ultimate premisses, this theory tells us what further rational beliefs, certain or probable, can be derived by valid argument from our direct knowledge. This involves purely logical relations between the propositions which embody our direct knowledge and the propositions about which we seek indirect knowledge. What particular propositions we select as the premisses of our argument naturally depends on subjective factors peculiar to ourselves; but the relations, in which other propositions stand to these, and which entitle us to probable beliefs, are objective and logical.

3. Let our premisses consist of any set of propositions h , and our conclusion consist of any set of propositions a , then, if a knowledge of h justifies a rational belief in a of degree, α , we say that there is a probability-relation of degree α between a and h .* (Keynes, 1921, p.4; italics added) Continuing with Keynes's position, he states that "There is nothing novel in the supposition that the probability of a theory turns upon the evidence by which it is supported; and it is common to assert that an opinion was probable on the evidence at first to hand, but on further information was untenable.

As our knowledge or our hypothesis changes, our conclusions have new probabilities, not in themselves, but relatively to these new premisses. New logical relations have now become important, namely those between the conclusions which we are investigating and our new assumptions; but the old relations between the conclusions and the former assumptions still exist and are just as real as these new ones. "(Keynes, 1921, p.7) and 11. In conclusion, the relativity of knowledge to the individual may be briefly touched on. Some part of knowledge—knowledge of our own existence or of our own sensations—is clearly relative to individual experience. We cannot speak of knowledge absolutely—only of the knowledge of a particular person. Other parts of knowledge—knowledge of the axioms of logic, for example—may seem more objective. But we must admit, I think, that this too is relative. What is self-evident to me and what I really know, may be only a probable belief to you, or may form no part of your rational beliefs at all. And this may be true not only of such things as my existence, but of some logical axioms also. Some men—indeed it is obviously the case—may have a greater power of logical intuition than others." (Keynes, 1921, p.17).

Keynes's conclusion is given below:

"But given the body of premisses which our subjective powers and circumstances supply to us, and given the kinds of logical relations, upon which arguments can be based and which we have the capacity to perceive, the conclusions, which it is rational for us to draw, stand to these premisses in an objective and wholly logical relation. Our logic is concerned with drawing conclusions by a series of steps of certain specified kinds from a limited body of premisses." (Keynes, 1921, p.18).

Therefore, a reading of chapters I and II of the TP, which clearly shows that Keynes considered evidence to be purely subjective and relative to the individual, I find Brekel's claims on pp.15-17 in Section 4.1 On Ignorance to have nothing to do with Keynes's logical theory of probability or his A Treatise on Probability. Brekel has simply put forth his own ideas which directly conflict with Keynes, just as Runde did some 36 years ago. Let us mention some of these claims:

“If, however, we interpret the I term in the above formalizations to refer to something like “the actual amount of the agent's relevant ignorance as judged from an omniscient perspective,” then relative Keynesian weight increases monotonically with the accrual of new evidence. “(Brekel,2025, p.15)

and

“Since the denominator of C_w remains constant over time, the total amount of relevant information, (i.e., $(K+I)$) is an unchanging quantity that exists independently of our estimations of ignorance. “(Brekel,2025, p.16)

and

“By treating ignorance as an objective measure, my interpretation keeps the amount of total relevant information constant despite any potential fluctuations in the estimated amount of ignorance. “(Brekel,2025, p.16)

and

“Treating ignorance as an actual, objective measure thus results in a substantively different conception of Keynesian weight than Runde's interpretation.

There exists one way to reconcile all the distinct considerations presented so far. Namely, we should interpret Keynesian weight as C_w , with the term for relevant ignorance (I) defined as the actual amount of relevant evidence that is unknown to the agent, as judged from an omniscient perspective. “(Brekel,2025, p.17).

The following claim by Brekel establishes that Brekel's reading of Keynes's TP amounts to a very poor reading of at most two chapters, chapters VI and XXVI, of the TP:

“At least in A Treatise on Probability, Keynes focuses on actual amounts of knowledge and ignorance rather than personal estimates of our knowledge and ignorance. The interpretation presented here better aligns with all relevant aspects of the source material.” (Brekel,2025, p.17).

In fact, Brekel(2025) has it upside down. Keynes focuses on the personal, subjective estimates of knowledge and ignorance, rather than the actual, objective amounts of knowledge and ignorance.

Brekel's last section,”4.2 Estimated and Actual Weight “, is intellectual irrelevant and has absolutely nothing to do with Keynes's analysis of evidential weight of the argument, V.

Keynes correctly analyzed V for an alert reader who combines chapters VI and XXVI of the TP.

5.0 CONCLUSIONS

What is new in Brekel is his claim that Keynes was not a subjectivist when it came to the discussion of the role of the assessment of evidence in his Boolean, relational propositional logic. This conclusion directly contradicts all of Keynes's discussions of the issue in chapters I and II of the TP. What is objective for Keynes in the TP is the logical, objective relation of probability, which was identical to Boole's objective, logical relation of probability, and the logical relation of evidential weight, to which Keynes applied the exact, same Boolean, relational, propositional logic. The logical relations of P and V connect the subjective evidence and conclusions in the h and a proposition. Ramsey argued that there was no such thing as a logical, objective probability relation that existed between the a and h propositions because Ramsey was completely ignorant of how Keynes had deployed Boole's relational, propositional logic. Brekel implicitly argues that there is no logical, objective weight relation existing between the subjective a and h propositions. Brekel, like Ramsey as regards to the P relation, argues that no such logical relation, V, exists, as it is actually a number, function, or variable.

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